~115 GeV and ~143 GeV Higgs mass considerations within the Composite Particles Model

Marko B. Popovic*

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Abstract

The radiatively generated Higgs mass is obtained by requiring that leading “divergences” are cancelled in both 2D and 4D. This predicts that the k=1 mode mass is $m_H \cong \frac{2}{3} m_t \cong 115 GeV$ whereas the k=2 mode is $\cong 143 GeV$. These findings are interpreted within the Composite Particles Model (CPM), [Popovic 2002, 2010], with the massive top quark being a baryon-like structure composed of 3 fundamental O quarks and the massive Higgs scalar being a color-neutral meson like structure composed of 2 fundamental O quarks. The CPM predicts that the Z mass generation is mediated primarily by a composite top – anti composite top whereas the Higgs mass is generated primarily by a fundamental O – anti O. The relationship [Popovic 2010] between top Yukawa coupling and strong QCD coupling, obtained by requiring that top – anti top scattering is zero at tree level at $\sqrt{s} \cong \mathcal{O}(M_Z)$, defines the Z mass. In addition, this relationship indirectly defines the electroweak symmetry breaking (EWSB) vacuum expectation value (VEV), Higgs mass and finally the Higgs Mass Zero Crossing (HMZC) scale [Popovic 2001; 2010] at which the effective Higgs mass squared becomes zero.

KEY WORDS: Dynamical Symmetry Breaking, Composite Particles Model, Renormalization, Higgs Mass.

*Please address all correspondence to:
Marko B Popovic, Physics Department, Worcester Polytechnic Institute
100 Institute Road, Worcester, MA 01609
Cell: 617 470 8198
Email: mpopovic@wpi.edu

1. Introduction

The Higgs scalar particle is the last missing ingredient of the Standard Model (SM) of particle physics. In this century two experimental results gave particularly dramatic hints on Higgs mass. The Large Electron Positron (LEP) particle accelerator near Geneva observed a number of suspicious events [1,2] in the vicinity of $115\, GeV/c^2$, at the center of mass energies a bit above $\sqrt{s} \cong 206\, GeV/c^2$, just before the accelerator was shut down in 2000. Now, after 10 years, the Fermi National Accelerator laboratory near Chicago reported excess in dijet invariant mass [3] in the vicinity of $144\, GeV/c^2$, at the $\sqrt{s} \cong 1.96\, TeV/c^2$ collision energies just before the planned shutdown in September 2011.

Here, I present a theoretical model that may explain either of these two "big finale" observations and it may assist the continued search at the Large Hadron Collider (LHC) at CERN near Geneva at 14 TeV energy, i.e. approximately 70,000 times larger than the scale of the strong nuclear reactions (200 MeV).

This model assumes radiative generation of the Higgs mass and cancellation of leftover leading "divergences" in both 2D and 4D within the standard SM scalar mass squared renormalization scheme. Two particularly interesting modes are obtained and then interpreted within the Composite Particles Model (CPM), [4, 5], that might closely mimic the effective SM broken phase.

According to CPM the massive top quark is a baryon-like structure composed of 3 fundamental O quarks and the massive Higgs scalar is a color-neutral meson-like structure composed of 2 fundamental O quarks. Note that the SM gauge anomaly cancellation is satisfied as the elementary top quark is exchanged with the original O quark.

The hierarchy problem is resolved via top-anti top interactions at Z mass energy, i.e. via defining relationship [5] between top Yukawa coupling and strong coupling constant g_{QCD}.

2. Electroweak symmetry breaking (EWSB) within the Composite Particles Model

The EWSB is a transition between two ground states. Here, instead of traditional Higgs and top quark fields in the unbroken massless phase the CPM has an auxiliary ('wannabe') Higgs field, with vanishing mass and quartic coupling terms, $m_H = 0$ and $\lambda = 0$, and an original quark field, O, with Yukawa coupling equal to $1/3$ of the top Yukawa coupling. The broken massive phase is expected to closely resemble SM with the original O quark field confined within the Higgs and top quark fields.

2.1. Zero VEV ground state

Here, I give condition that renormalized auxiliary field stays massless in leading order in 2D and 4D.

2.1.1. Mass renormalization without propagating Higgs

Leading 2D SM renormalized mass squared logarithmic "divergences", *without* propagating Higgs are

$$\propto\ w_{2D}\frac{g_Y^2+3g_W^2}{4} - \sum\left(\frac{n_C}{3}\right)3g_f^2 \tag{1}$$

with summation over all elementary fermions and $n_C = 3\ (1)$ for quarks (leptons), see e.g. [6, 5].

In 4D, leading SM renormalized mass squared quadratic "divergences", *without* propagating Higgs are

$$\propto w_{4D}\frac{g_Y^2+3g_W^2}{4}-\sum\left(\frac{n_C}{3}\right)2g_f^2 \qquad (2)$$

Factors $w_{2D}=1$and $w_{4D}=2/3$ are ratios of massless over massive gauge boson polarization degrees of freedom in 2D and 4D. If couplings are identical in both 2D and 4D and "divergences" are cancelled in both 2D and 4D*,* a dominant Yukawa coupling squared, g_f^2, equals

$$g_f^2=\frac{g_Y^2+3g_W^2}{12}\,. \qquad (3)$$

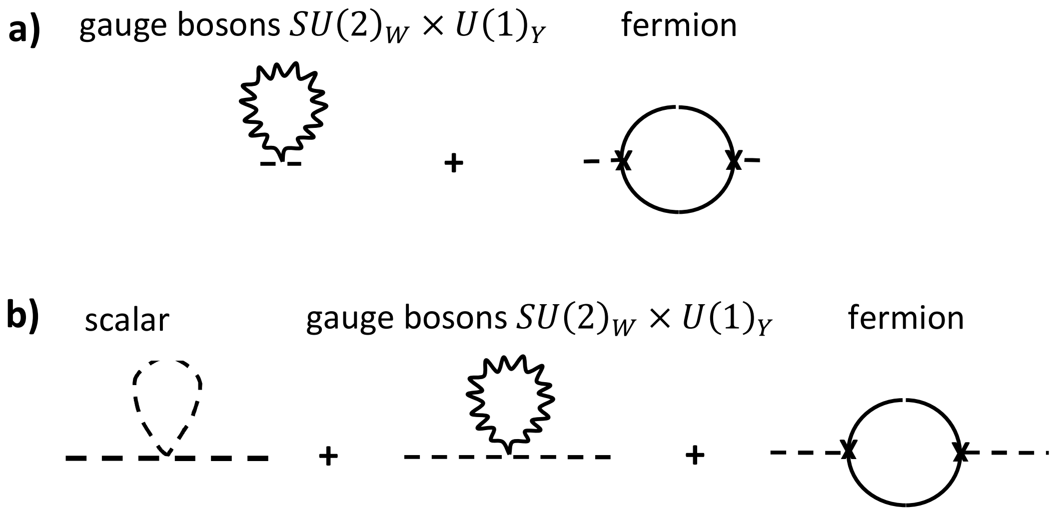


Figure 1 a) Auxiliary scalar field mass renormalization without scalar propagation. b) Auxiliary scalar field mass renormalization with propagating scalar.

2.1.2. Mass renormalization with propagating Higgs

In 2D, leading SM renormalized mass squared logarithmic "divergences", *with* propagating Higgs are

$$3\lambda+w_{2d}\frac{g_Y^2+3g_W^2}{4}-\sum\left(\frac{n_C}{3}\right)3g_f^2\ . \qquad (4)$$

In 4D, leading SM renormalized mass squared quadratic "divergences", *with* propagating Higgs are

$$\lambda+w_{4d}\frac{g_Y^2+3g_W^2}{4}-\sum\left(\frac{n_C}{3}\right)2g_f^2 \qquad (5)$$

From Equ (4, 5) dominant quark Yukawa coupling squared and scalar field quartic coupling equal

$$g_f^2=\left(w_{4d}-\frac{w_{2d}}{3}\right)\frac{g_Y^2+3g_W^2}{4}\ \text{ and }\ \lambda=\left(w_{4d}-2\frac{w_{2d}}{3}\right)\frac{g_Y^2+3g_W^2}{4} \qquad (6)$$

Hence, for expected $w_{2d}=1$ (longitudinal) and $w_{4d}=2/3$ (2 massless transversal over 3 massive total)

$$g_f^2 = \frac{g_Y^2+3g_W^2}{12} \text{ and } \lambda = 0. \quad (7)$$

Therefore, the same result is obtained with and without Higgs propagation, Equ (3, 7).

2.1.3. Yukawa coupling defined with low energy gauge couplings

If Equ (7) or Equ (3) is satisfied for the low energy SM values of $U(1)_Y \times SU(2)_W$ couplings g_Y, g_W then

$$g_f \cong \frac{g_t}{3}. \quad (8)$$

However, no SM fermion has this Yukawa coupling and Higgs quartic coupling is not zero.

2.2. Non-zero VEV ground state

I address the condition that radiatively generated scalar mass is in agreement with renormalization group equations in 2D description. Why is 2D description important? As is demonstrated by the lattice arguments, e.g. see [7], the non-Abelian gauge fields carry charge that causes their propagation to mimic the 1-space dimensional flux providing confinement between static charges. Hence, the 2D considerations here are thought of as consequence of non-Abelian gauge fields' dynamics in regular 4D.

As first emphasized by Nambu, in simplest models with dynamical mass generation [8-10] and top condensate [11-14] EWSB, one may expect the Higgs mass on the order of 2 top quark masses. Hence, if top quark is exchanged with O quark one might expect the Higgs mass on the order of $\frac{2}{3}m_t$ [4, 5].

2.2.1. <u>Transition at low energies: 2D consideration</u>

Let us consider EWSB, where "original" fields and parameters, abruptly change across phase transition and Higgs mass is $\frac{2}{3}m_t$. The effect of "transition" can be expressed by an unknown parameter A

$$g_f \cong \frac{g_t}{3} \to Ag_f \cong A\frac{g_t}{3}, \quad (9a)$$

$$m_H = 0 \to m_H = \frac{2}{3}m_t \text{ or } \lambda = 0 \to \lambda = \frac{m_H^2}{v_{EW}^2} = \frac{4m_t^2}{9v_{EW}^2} = \frac{2g_t^2}{9}. \quad (9b)$$

Imposing the cancelation of leading "divergences" in 2D, Equ (4), with help of Equ (3, 7,9b) leads to

$$3\lambda + \frac{g_Y^2+3g_W^2}{4} = \frac{6g_t^2}{9} + 3g_t^2\left(\frac{1}{3}\right)^2 = 3g_t^2\left(\frac{1}{3}\right)^2 A^2 \quad (10)$$

$$\Rightarrow A^2 = 3 \text{ or } g_f \cong \frac{g_t}{3} \to \sqrt{3}g_f \cong \frac{g_t}{\sqrt{3}}. \quad (11)$$

However, not all "divergences" should cancel because Higgs mass is expected to be radiatively generated. Hence, the physical top Yukawa coupling is $g_t \cong 1$ whereas $\frac{g_t}{\sqrt{3}} \cong \frac{1}{\sqrt{3}}$ is the factor defining the fermion loop contribution to cancelation of just a part of leading "divergences". I explain that next.

2.2.2. Effect of radiatively generated Higgs mass: 2D calculation

I split dominant fermion loop into two terms; the first term (x-term) contributes to the cancellation of leading "divergences" in Equ (4) and the second term (y-term) equals to the radiatively generated Higgs mass. Two terms add to one, i.e. $x + y = 1$ and the single color 2D fermion loop is exactly solved [5] as

$$2D\ fermion\ loop = \frac{kg^2}{\pi}. \tag{12}$$

This result is obtained with similar techniques as the fermion loop in the Schwinger model [15]. However, in contrast to the Schwinger model this is a composite scalar and not a gauge boson. Hence, if "fermion loop" is indeed identified as a "*single* fermion loop" in a pure 2D calculation (i.e. no 4D spin) one would expect k=1. However, to make the connection with 4D, one should expect a factor of k=2 here, due to the scalar nature of the propagator, as fermion spins may point inward or outward (when 4D spin is also considered). Hence, I leave an explicit dependence on the relevant phase space parameterized with k. Finally, in 2D, the mass singularities in the propagator should exist for multiple integer values [7, 15]. The "*multi*-fermion loop" interpretation of $k = 1, 2$ modes and reason for $g = g_t$ are provided in 2.2.3.

The obtained system has three unknowns, x, y and λ, and three equations,

$$3\lambda + \frac{g_Y^2+3g_W^2}{4} = 3g_t^2 x\ ,\ \ x + y = 1 \quad \text{and} \quad \lambda = \frac{kg_t^2}{\pi} y\,, \tag{13}$$

leading to an unique solution

$$3\lambda + \frac{g_Y^2+3g_W^2}{4} = 3g_t^2\left(1 - \frac{\lambda\pi}{kg_t^2}\right) \rightarrow \sqrt{\lambda} = \sqrt{\frac{g_t^2 - \frac{g_Y^2+3g_W^2}{12}}{1+\frac{\pi}{k}}}\,, \tag{14}$$

$$\Rightarrow\ \ m_H = \sqrt{\frac{6m_t^2 - M_Z^2 - 2M_W^2}{3\left(1+\frac{\pi}{k}\right)}}. \tag{15}$$

The above calculation, see [5], is self consistent as the Higgs mass in the Higgs loop propagator (within piece proportional to x) is identical to radiatively generated Higgs mass (within piece proportional to y).

For the world average top quark mass, $m_t = 173.1 \pm 1.3\ GeV$, I obtain

$$m_H = \begin{cases} 113.0\ \pm 1.0\ GeV\ for\ \ k = 1\ \rightarrow\ y = \mathbf{0.669},\ x = 0.331 \\ 143.4\ \pm 1.3\ GeV\ for\ \ k = 2\ \rightarrow\ y = \mathbf{0.539},\ x = 0.461 \end{cases}. \tag{16}$$

Hence, for the k=1 mode, the parameterized effective top Yukawa coupling, specific to cancellation of part of the "divergences" is equal to $g_t\sqrt{0.331} \cong \frac{1}{\sqrt{3}}$, exactly as anticipated with Equ (11)! And predicted Higgs mass is roughly as originally anticipated, i.e. $m_H \cong \frac{2}{3} m_t = 115.4\ GeV$!

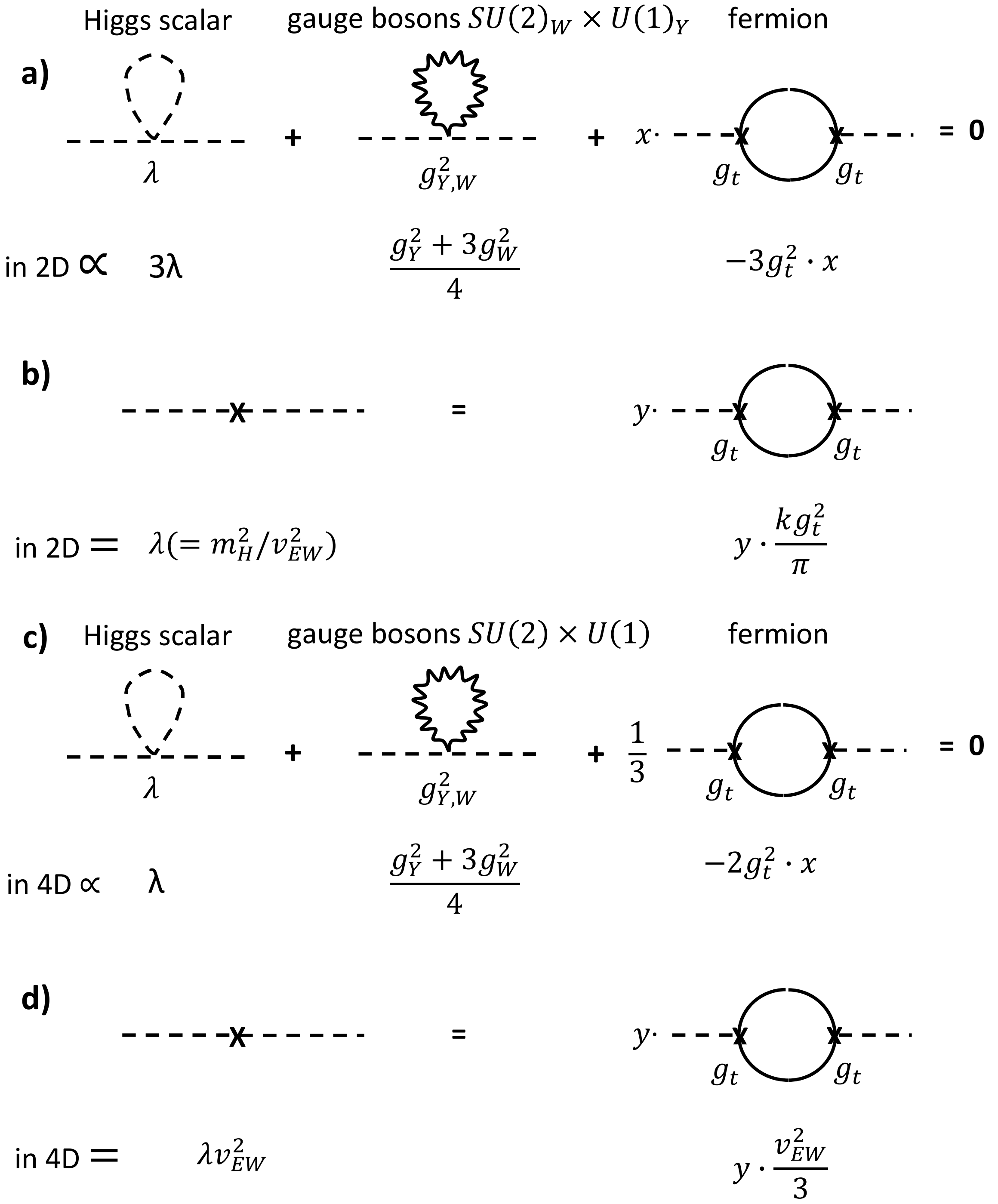


Figure 2 a) Self-consistent cancelation of "divergences" and b) radiative generation of Higgs mass in 2D. c) Self – consistent cancelation of "divergences" in 4D and d) radiative generation of Higgs mass in 4D.

2.2.3. Interpretation of 2D k=1 mode from O-anti O condensate

The Higgs mass squared in Equ (13) within 2D k=1 mode may be rewritten as

$$y\frac{g_t^2}{\pi} \cong \frac{2}{3}\frac{g_t^2}{\pi} = 6\frac{\left({g_t}/{3}\right)^2}{\pi} = n_S n_C \frac{\left({g_t}/{3}\right)^2}{\pi} \tag{17}$$

where $n_S = 2$ and $n_C = 3$ are spin and color contributions to phase space respectively and where $\cong {g_t}/{3}$ is O Yukawa coupling. Hence, 2D k=1 Higgs appears to be generated via O -anti O condensate.

2.2.4. Interpretation of 2D k=2 mode from O-anti O and top-anti top condensate

The Higgs mass squared in Equ (13) within 2D k=2 mode may be rewritten as

$$y\frac{2g_t^2}{\pi} \cong 0.539\frac{2g_t^2}{\pi} = 0.539 \cdot 2 \cdot 9\frac{\left({g_t}/{3}\right)^2}{\pi} = 9.702\frac{\left({g_t}/{3}\right)^2}{\pi}. \tag{18}$$

Now, I hypothesize that top – anti-top condensate within CPM may also contribute to 2D k=2 Higgs as

$$n_S n_C \pi^2 \frac{\left({g_t}/{3}\right)^2}{\pi} \tag{19}$$

where factor π^2 is the phase space of 2 transversal O – anti O condensates with axial $(0,\pi)$ symmetry.

Here, I expect that weighted sum of 2D k=1 mode and contribution from the top –anti top condensate should be equal to 2D k=2 Higgs mass and parameterized with an angle α,

$$\cos^2\alpha \cdot n_S n_C + \sin^2\alpha \cdot n_S n_C \pi^2 \cong 9.702 \;\Rightarrow\; \cos^2\alpha = 0.930 \tag{20}$$

2.2.5. 4D matching

Next, I address k=1 mode in 4D. Again the "fermion loop" is split into x and y components

$$\lambda + \frac{g_Y^2 + 3g_W^2}{4} - 2g_t^2 x = 0 \tag{21}$$

and the radiatively generated Higgs mass is product of y, 4D single color fermion loop, and $\cos^2\alpha$,

$$\lambda v_{EW}^2 = y \cdot \text{4D fermion loop} \cdot \cos^2\alpha. \tag{22}$$

If each color contributes the same amount to VEV squared then

$$\lambda = y \cdot \frac{1}{3} \cdot \cos^2\alpha = (1-x) \cdot \frac{1}{3} \cdot \cos^2\alpha \;\Rightarrow\; x = 1 - \frac{3}{\cos^2\alpha}\lambda\ . \tag{23}$$

Substituting expression for x back into Equ (21), for angle α as in Equ (20), gives

$$\sqrt{\lambda} = \sqrt{\frac{2g_t^2 - \frac{g_Y^2+3g_W^2}{4}}{1 + \frac{6}{\cos^2\alpha} g_t^2}} \;\Rightarrow\; m_H = \sqrt{\frac{4m_t^2 - M_Z^2 - 2M_W^2}{1 + \frac{12}{\cos^2\alpha}\frac{m_t^2}{v_{EW}^2}}} \cong 115.6\ GeV. \tag{24}$$

with 4D $x = 0.289, y = 0.711$ and scalar mass differs only $0.2\ GeV$ from the CPM k=1 prediction.

However, if 2D k=1 mode's $x = 0.311$ is enforced in 4D then from Equ (21)

$$\lambda + \frac{g_Y^2+3g_W^2}{4} - 2g_t^2 x = 0 \Rightarrow m_H = \sqrt{4 \cdot x \cdot m_t^2 - M_Z^2 - 2M_W^2} = 138.1\ GeV. \qquad (25)$$

This result was obtained by Popovic [4, 5] and it should be compared with a "long lived" SM solution [4, 5] where both dimensionless parameters of the Higgs potential tend towards 0 at $\sim$ the Planck scale.

By adding top – anti top contribution to 4D O – anti O result, Equ (25), I obtain

$$m_H = 138.1\ GeV \cdot \sqrt{1 + \tan^2\alpha} = 143.2\ GeV \qquad (26)$$

with the same angle as in Equ (20, 22); the scalar mass differs only $0.2\ GeV$ from the CPM k=2 prediction.

It would be worthwhile investigating whether the Fermi lab findings [2], i.e. Gaussian excess centered at 144 GeV dijet invariant mass, originate indirectly from (1) processes involving W boson, CPM O and bottom quarks (sum of their masses is $M_W + \frac{m_t}{3} + m_b \cong 142.5\ GeV$) or directly from (2) decay of k=2 mode 143.4GeV Higgs interacting with both O –anti O (96.4%) and top –anti top (3.6%) condensates.

3. *The Bose-Einstein distribution applied to condensates mixing*

Higgs field, Φ, may be shared by O -anti O, top – anti top condensate, and maybe something else

$$\Phi = c_2\, \mathrm{O}\overline{\mathrm{O}} + c_6\, t\bar{t} + \cdots \qquad (27)$$

where coefficients $c_2, c_6, \cdots$ (subscripts are chosen to remind that this is CPM model) are relative contributions normalized to one. By applying the Bose-Einstein distribution with assumed values

$$\hbar\omega_2 \cong m_{o\bar{o}}c^2 \cong kT\ ,\ \hbar\omega_6 \cong m_{t\bar{t}}c^2 \cong 3\, m_{o\bar{o}}c^2\ , \qquad (28)$$

$$\Rightarrow c_2 = \frac{\frac{1}{e^1-1}}{\frac{1}{e^1-1}+\frac{1}{e^3-1}+\cdots} = \frac{0.58197}{0.58197+0.05239+\cdots} \le 0.91741 \qquad (29)$$

with $m_{t\bar{t}} = 2m_t$ and $m_{\mathrm{O}\overline{\mathrm{O}}} = \frac{2}{3}m_t$ on the lines of top condensate models as emphasized by Nambu [14]. The mass contribution to Higgs field from each condensate type can be expressed as

$$m_H = c_2 \cdot 2\frac{m_t}{3} + c_6 \cdot 6\frac{m_t}{3} + \cdots \qquad (30)$$

$$\Rightarrow\ m_H \ge 0.91741 \cdot 2\frac{m_t}{3} + (1 - 0.91741)6 \cdot \frac{m_t}{3} \text{ or } m_H \ge 134.5\ GeV. \qquad (31)$$

Clearly this result is dependent on kT . For $kT \cong M_Z c^2$ I obtain $c_2 \le 0.94477$ and $m_H \ge 128.1\ GeV$. However, for the pole-consistent solutions, $kT \cong m_H c^2$, I obtain the smallest possible Higgs mass as

$$c_2 \le 0.89484 \text{ and } m_H \ge 139.7\ GeV, \qquad (32)$$

in a close agreement with k=2 Higgs mass addressed above. This result again corresponds to

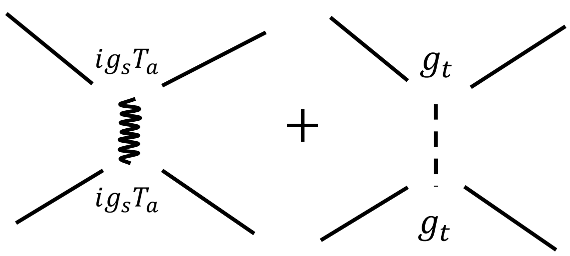
$$\cos^2\alpha \cong [1-a(1-c_2)]^2 = 0.931 \text{ for } a = \frac{1}{3} = \frac{m_{o\bar{o}}}{m_{t\bar{t}}}. \quad (33)$$

4. Z mass and top – anti top long range massless mode

Here, I investigate whether the SM $\bar{t}t$ channel at energy of Z pole mass is repulsive or attractive.

Consider the $\bar{t}t$ scattering in the Euclidean space and ignore chiralities of the incoming and outgoing particles while assuming that left and right handed tops are equally represented within particle and antiparticle solution. The main interaction channels at tree level are gluon and Higgs exchange. The weak interactions are absent as interacting particles have opposite chiralities and the hypercharge interactions are zero due to the equal sharing conjecture introduced above.

I now assume that strong QCD interactions proportional to $-g_{QCD}^2\, T_{aij}T_{akl}$, where $a = 1 \dots 8,\ i,j = 1,2,3$ and summation over repeated indices is implied, are exactly *balanced* with the Yukawa forces due to the virtual Higgs particle exchange proportional to g_t^2 as a condition for the loose bound state, see Fig 3.



$$\frac{1}{3}g_s^2 \times 2 \qquad -g_t^2 \qquad = \mathbf{0} \rightarrow \alpha_s = \frac{3}{2}\alpha_t = \frac{3g_t^2}{8\pi} = 0.1181 \pm 0.0018$$

World average $\alpha_s \cong 0.1184 \pm 0.0007$

Figure 3 Finely balanced interplay between the QCD gluon and Higgs scalar mediated top anti-top interactions.

Hence, the back of the envelope calculation at tree level suggests

$$2\frac{1}{2}\frac{2}{3}g_{QCD}^2 = g_t^2 \ \ or \ \ \alpha_S = \frac{3}{2}\frac{g_t^2}{4\pi} \text{ at Z mass} \quad (34)$$

where I used $(T^a)_{ij}(T^a)_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{kj} - \frac{1}{N}\delta_{ij}\delta_{kl}\right)$ for $SU(N)$ groups, see for example [16], where $N = 3$. For the *colorless* composite Higgs the expression in bracket equals $\frac{2}{3}$, and additional factor of 2 in Equ (34) corresponds to two transversal gluon polarizations. ***QED***

The result in Equ (34) is in an excellent agreement with the standard estimate of the strong running coupling constant [17, 18]. Equ (34) predicts $\alpha_s = 0.1181 \pm 0.0018$ given the world average top quark mass $m_t = 173.1 \pm 1.3\ GeV$ where uncertainty is therefore solely due to the top quark mass uncertainty. This can be compared with the current world average value $\alpha_s \cong 0.1184 \pm 0.0007$ at $s = M_Z^2$ [17, 18].

Even if the equal distribution assumption is ignored and hypercharge interactions are taken into account that would change the above result only on the order $\frac{3}{2} Y_L Y_R \frac{g_Y^2}{g_t^2} = \frac{3}{2}\frac{1}{6}\frac{2}{3}\frac{g_Y^2}{g_t^2} \sim 2\%$.

Interpretation of this result is that it takes zero energy to orient the top – anti top at Z scale; these excitations are the long range massless bosons that couple to massless Z and give Z mass. That is not the case with original field O as $g_{QCD}^2 \gg \frac{3}{2}\left(\frac{g_t}{3}\right)^2$. Hence Equ (34) is likely the underlying principle defining the EWBS scale, spanning vast energies in a natural fashion and removing the hierarchy problem.

5. Summary

I review earlier CPM results [4, 5] and introduce the concept of condensates mixing. The CPM predicts the Z mass generation mediated primarily by composite top – anti composite top interactions whereas the physical Higgs mass is generated primarily by fundamental O – anti fundamental O interactions. The relationship [5], Equ (34), between top Yukawa coupling and strong QCD coupling defines the Z mass and hence indirectly the EW VEV, Higgs mass and finally the HMZC scale, see [19, 5] at which the effective Higgs mass squared transitions from positive to negative values. The HMZC≅EWSB [5] if SM is the effective theory at low energies and if the Universe dynamics was never dominantly tachyonic.

The radiatively generated k=1 Higgs mass is expected to be $m_H \cong \frac{2}{3} m_t \cong 115\ GeV$ whereas k=2 mass is $\cong 143\ GeV$. These predictions are confirmed by cancellation of leftover leading "divergences" in both 2D and 4D. The hierarchy problem is resolved by defining relationship Equ (34). Similarly the vacuum energy problem might be resolved as, due to dynamic nature of process, it is probably not necessary to assume ground state which is uniform and homogenous in the entire 4D space [4, 5].

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References

[1]CERN Press release
<http://press.web.cern.ch/Press/PressReleases/Releases2000/PR08.00ELEPRundelay.html> (2000)
[2] P. Renton, Nature 428, 141-144 (2004)
[3] T. Aaltonen et al., arXiv:1104.0699v1 [hep-ex] 4 April 2011
[4] M. B. Popovic, Harvard University Theory Physics, HUTP-02/A012, hep-ph/0204345 (2002)
[5] M. B. Popovic, arXiv:1009.5054 [hep-ph] 26 Sep 2010
[6] M. Veltman, Acta Phys. Pol. B12, 437. (1981)

[7] J. Zinn-Justin, “Quantum Field Theory and Critical Phenomena” 2nd ed., Clarendon Press, Oxford (1993).
[8] Y. Nambu and G. Jona-Lasinio, Physical Review 122: 345–358. (1961)
[9] Y. Nambu and G. Jona-Lasinio, Physical Review 124: 246–254. (1961)
[10] Vaks V. G., Larkin A. I., “On the application of the methods of superconductivity theory to the problem of the masses of elementary particles,” Zh. Eksp. Theor. Fiz. 40 282-285. (1961) [English transl. : Sov. Phys. JETP 13 192-193. (1961)]
[11] C. T. Hill, FERMILAB-PUB-80-097-T, Dec 1980. 44pp. Published in Phys.Rev.D24, 691. (1981)
[12] V.A. Miransky, M. Tanabashi, and K. Yamawaki, DPNU-89-08, Jan 1989. 20pp. Published in Mod.Phys.Lett.A4:1043. (1989)
[13] W. A. Bardeen, C. T. Hill, and M. Lindner, FERMILAB-PUB-89-127-T, Jul 1989. 43pp. Published in Phys.Rev.D41, 1647. (1990)
[14] Y. Nambu, in Z. Adjduk, S. Pokorski, and A. Trautman. Proceedings of the Kazimierz 1988 Conference on New Theories in Physics. XI International Symposium on Elementary Particle Physics. pp. 406–415. (1989)
[15] J. Schwinger, Phys. Rev. 128, 2435. (1962)
[16] M. E. Peskin and D. V. Schroeder, An Introduction to Quantum Field Theory, Addison Wesley Publishing Company. (1995)
[17] C. Amsler et al. (Particle Data Group), Physics Letters B667, 1 (2008) and 2009 partial update for the 2010 edition (URL: http://pdg.lbl.gov). (2010)
[18] S. Bethke, arXiv:0908.1135 [hep-ph]. (2009)
[19] M. B. Popovic, Boston University High Energy Physics, BUHEP-01-15, hep-ph/0106355 (2001)