Global Motion Control and Support Base Planning

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Abstract **– Advanced humanoid robots capable of operating in complex 3-D environments will likely utilize an on-line optimization strategy where joint accelerations are varied to achieve whole-body postural balance. To this end, we propose one such strategy that optimizes global body parameters such as spin angular momentum and body principal angles, or the angles between the inertia tensor principal axes and the lab frame axes. This optimization strategy is easily combined with other optimization objectives (e.g. maximal efficiency) subject to physical constraints such as requiring that the ZMP operates within the support base. To deal with Bellman's "curse of dimensionality" we suggest, in parallel, two computational simplifications that may make the optimization problem tractable and easily implemented on today's humanoid robots. Finally, we address the problem of support base planning during ground and aerial locomotory phases. We propose novel on-line strategies for robust coordination of interacting limbs compatible with the proposed optimization strategy.**

Index Terms: humanoid, control, angular momentum, global motion control, support base planning

I. INTRODUCTION

Humanoid robots that can truly mimic human movement patterns have not yet been advanced and necessitate fundamental advances in hardware and control design [1,2]. Robust and adaptive control of autonomous humanoids is indeed a difficult problem. For humanoid robots to be practical and useful they should be capable of performing novel tasks within the same complex environment in which humans operate. Unlike industrial manipulators, it is neither possible nor meaningful to track a small set of predetermined joint trajectories [3]. Moreover, without an attachment base securely bolted to the ground, postural balance is a primary control task for humanoid robots [4]. Still further, the physical problem is not forgiving of bad control policies; for some situations even small errors in system state may have catastrophic consequences.

General, precise and practical formulation of postural stability as a control problem remains elusive [6,7]. Many studies of repetitive motions utilize return map analyses *a posteriori* to tackle stability in a plant/task/condition (and particular target trajectories) specific manner. However, there are many potential plants, tasks and transitions as well as external world conditions. Hence, it seems beneficial, if not necessary, for an online controller to exploit an on-line optimization strategy where joint accelerations are varied to achieve whole-body postural balance.

Time-local optimizations strategies for postural balance have been previously proposed. Studies relied on placing the center of mass (CM) ground projection as close as possible to the innermost point of the support base to ensure stability [8,9]. As known for centuries, stability is satisfied for the static case when the zero moment point, or ZMP, is positioned just beneath the CM [10,11]. Along similar lines, the ZMP distance from the support base boundary was employed as a stability margin [12]. Finally, recent studies explored the horizontal component (orthogonal to gravity) of angular momentum about the CM and the associated moment as indicators of postural stability [13,14]. However, by themselves none of these proposed stability metrics guarantee stability [15]. Moreover, not all of the proposed metrics are supported by biomechanical observations. During a walking gait cycle, the CM crosses the support base only during a short segment of the double support phase. Still further, the ZMP spans nearly the entire foot length during the single support phase. Finally, the angular momentum is highly regulated in walking but is less regulated for activities like a twirling hula-hoop motion or balancing on a tight rope [14-17].

In Section II we propose a novel angular momentum-based, optimization strategy expressed in terms of global quantities, or quantities representing whole body translational and rotational dynamics. In Sections II and III we introduce the Global Motion Control (GMC) framework suitable for the control of high level integrated quantities. Though the proposed optimization strategy and ZMP location may be expressed only in terms of global quantities, we extend our control framework from global state to joint space to effectively control balance as well as additional performance objectives.

Due to the high dimensionality of the plant being controlled, the general optimization problem has to be simplified so that the controller may operate in real time. One approach is to reduce the dimension of joint state space by utilizing motion primitives [18]. Another approach is a hard-coded hierarchical or prioritized control [19,8,9]. Here the controller, local in time, first satisfies the most important task and then continues down a predetermined priority list. Although this method fails to address situations when a real compromise between tasks needs to be made, it can be very efficient and operate in the context of a real time controller. This method was recently utilized for non-contact limb balancing [20]. In Section III we propose a soft-coded variant of this approach in which the priority list is decided in an automatic, time-local fashion. We also propose a non-prioritized approach for the control cases for which the cost function can be expressed in a particularly simple form such that an analytic solution is possible. In Section III we illustrate the later method by enforcing linear dynamics and truncating all control variables to linear terms in joint jerks. In Section IV we propose an approximate GMC-based time-local metric that may be utilized for support base planning during ground and flight locomotory phases. Finally, in Section V we condense our findings and point to future research directions.

II. GLOBAL MOTION CONTROL (GMC)

A *From Spin Regulation to GMC*

Biomechanical investigations have determined that for normal, level-ground human walking, spin angular momentum, or the body's angular momentum about the CM, remains small throughout the entire walking cycle, including both single and double support phases [13, 21, 14, 22]. In these investigations, a morphologically realistic human model and kinematic gait data were used to estimate spin angular momentum at self-selected walking speeds. Walking spin values were then normalized by dividing by body mass, total body height, and walking speed. The resulting dimensionless spin was surprisingly small. Throughout the gait cycle, none of the three spatial components ever exceeded 0.02 dimensionless units [14]. If the human body is approximated with a uniform rod of length H then the minimal value of normalized angular momentum in the moment of fall caused by infinitesimally perturbed upright posture, would be $\frac{\pi}{24V}\sqrt{\frac{3gH}{2}} \approx 0.5$ for V=1.3 m/s. The gait studied in [14] would be within the 4% of this *ad hoc* angular momentum stability margin.

To determine the effect of the small, but non-zero angular momentum components on whole body angular excursions, the whole body angular velocity vector and the corresponding angular excursions were computed, or

$$\vec{\omega} = \dot{\vec{\theta}} = \vec{I}^{-1}(\vec{r}_{CM})\vec{L}(\vec{r}_{CM}), \quad \vec{\theta}(t) = \int_{-\infty}^{t} \vec{\omega}(t^*)\, dt^* + C \qquad (1)$$

respectively, where $\vec{I}(\vec{r}_{CM}) = \sum_{i=1}^{N} \vec{I}_i(\vec{r}_{CM})$ is the whole body inertia tensor about the CM and C is an integration constant determined through an analysis of boundary conditions [14]. Although there is no unique relationship between posture and the whole body angular excursion, the vector defined in (1) can still be accurately viewed as the rotational analog of the CM position vector, i.e. $\vec{r}_{CM}(t) = \int_{-\infty}^{t} \vec{v}_{CM}(t^*)\, dt^* + D$. The results of these analyses show that the maximum whole body angular excursions within sagittal (<1°), coronal (<0.2°), and transverse (<2°) planes are negligible [14].

B *GMC PD control law for global state*

Consider a simple PD control law relating whole body[1] angular excursions, $\vec{\theta}$, spin angular momentum, $\vec{L}(\vec{r}_{CM})$, CM position, $\vec{r}_{CM}$, and CM momentum, $\vec{p}$, with desired whole body net moment about the CM, $\vec{\tau}_{des.}(\vec{r}_{CM})$, and net CM force, $\vec{F}_{des.}$, or

$$\vec{\tau}_{des.}(\vec{r}_{CM}) = \dot{\vec{L}}_{tar}(\vec{r}_{CM}) - \tilde{a}\,\Delta\vec{\theta} - \tilde{b}\,\Delta\vec{L}(\vec{r}_{CM}), \qquad (2a)$$

$$\vec{F}_{des.} = \dot{\vec{p}}_{tar} - \tilde{c}\,\Delta\vec{r}_{CM} - \tilde{d}\,\Delta\vec{p} \qquad (2b)$$

In (2a) $\Delta\vec{\theta} = \vec{\theta} - \vec{\theta}_{tar.}$, $\vec{\theta}_{tar.}$ is the target body angular excursion; $\Delta\vec{L}(\vec{r}_{CM}) = \vec{L}(\vec{r}_{CM}) - \vec{L}_{tar.}(\vec{r}_{CM})$, $\vec{L}_{tar.}(\vec{r}_{CM})$ is the target spin angular momentum; and positive definite 3 by 3 matrices $\tilde{a}$ and $\tilde{b}$ are rotational stiffness and damping coefficients respectively. Analogously, in (2b) $\Delta\vec{r}_{CM} = \vec{r}_{CM} - \vec{r}_{CM\,tar.}$, $\vec{r}_{CM\,tar.}$ is the target CM position; $\Delta\vec{p} = \vec{p} - \vec{p}_{tar.}$, $\vec{p}_{tar.}$ is the target CM momentum; and positive definite 3 by 3 matrices $\tilde{c}$ and $\tilde{d}$ are stiffness and damping coefficients respectively. For practical purposes, instead of whole body angular excursions, which are not directly measurable quantities, one may consider using whole body principal angles defined by the relative orientations of the principal axes of the whole body inertia tensor with respect to the non-rotating lab frame axes.

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[1] The term 'whole body' is used to denote the body plus any attached or carried weight (e.g. backpack).

The control designer may choose the diagonal form for matrices $\tilde{a}$, $\tilde{b}$, $\tilde{c}$ and $\tilde{d}$ and also set some of the diagonal elements to zero. For a humanoid robot in steady state walking, one may anticipate that the desired whole body angular excursion and the spin angular momentum would both be set to zero and the rotational stiffness and damping coefficients would then be adjusted to achieve a desired system response. Also, $\vec{r}_{CM\,tar.} = \vec{r}_{CM}(t=0) + \vec{p}_{tar.}t/M$ with $\vec{p}_{tar.} = const.$ for $t \in (0,T)$ where T is chosen period of time.

The novelty of (2) is that it employs the rotational analog of the CM position and that it unifies all global quantities into one simple proportional derivative (PD) control law. We name this relationship the Global Motion Control (GMC) PD law. Similar to (2b), control of the CM position has been addressed by [23, 20] and joint linear and angular momentum control has been addressed by [24,25].

C *Stability metrics and GMC PD control law*

The GMC PD law does not communicate *a priori* any type of stability metric. By definition it is only a tool for controlling the global state variables. However, if specific terms, like $\Delta\vec{L}(\vec{r}_{CM})|_{hor.}$ with $\vec{L}_{tar.}(\vec{r}_{CM}) = 0$, denote the stability metric, then equation (2) supplies important guidance for postural stability. Consider steady state walking — if rotational stiffness and damping coefficients ideally reflect the nature of the control problem then the actual moment, $\vec{\tau}_{act.}(\vec{r}_{CM})|_{hor.}$, of the opposite sign from $\vec{\tau}_{des.}(\vec{r}_{CM})|_{hor.}$ should be considered destabilizing. However even the stabilizing $\vec{\tau}_{act.}(\vec{r}_{CM})|_{hor.}$ may not guarantee actual postural stability unless it is the right magnitude.

We now generalize the angular momentum metric to include angular excursions as well. In addition we add some level of sensitivity to the external forcing and task dynamics.

Consider first the dynamics in the CM frame. In the non-inertial CM frame the body segments experience inertial forces in addition to external forcing. As we show next, CM non-inertiality embedded in (2b), i.e. $\vec{p}_{tar.} \neq const.$, may be coupled to rotational dynamics, (2a). The constant lab frame target acceleration may be thought to define a new effective gravity vector in the CM frame, Fig. 1a. The effect is identical to $\dot{\vec{p}}_{tar.} = 0$ when the plant experiences constant and uniform non-ground-reaction-forces (non-GRF) external forcing (e.g. wind), Fig. 1b. In direct analogy with regular upright posture, i.e. zero non-GRF external forcing and zero target acceleration, we propose that target angular excursion and momentum components orthogonal to the effective gravity vector should be set to zero. Therefore, (2a) may be now expressed in decoupled form as

$$\vec{\tau}_{des.}(\vec{r}_{CM}) = \dot{\vec{L}}(\vec{r}_{CM}) - \tilde{a}_{H}\vec{\theta}|_{eff.hor.} - a_V\Delta\vec{\theta}|_{eff.ver.} - \tilde{b}_H \vec{L}(\vec{r}_{CM})|_{eff.hor.} - b_V\Delta\vec{L}(\vec{r}_{CM})|_{eff.ver.} \qquad (3)$$

where the effective gravity vector is defined with $\dot{\vec{p}}_{tar.}$ and the slowly varying component of the non-GRF external forcing. The rotational stability measure is now represented by deviation of actual angular excursion and momentum from their most stable global state configuration

$$\vec{\theta}_{tar.}|_{eff.hor.} = 0 \text{ and } \vec{L}_{tar.}(\vec{r}_{CM})|_{eff.hor.} = 0 . \qquad (4)$$

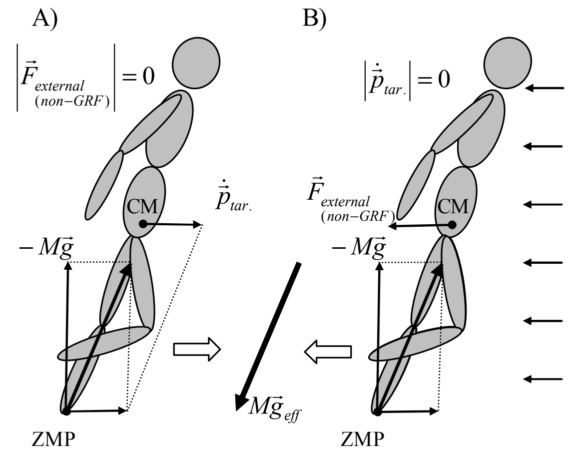

Fig. 1 Effective gravity vector for constant or slowly varying a) acceleration and b) non-GRF external forcing.

Finally, all translational terms embedded in the GMC PD law, (2b), may be also thought of as indirect stability measures. However, they are more directly related to pure motion planning than to time-local postural stability measures. In other words, one is clearly more stable if the target CM trajectory doesn't require body collisions with walls or falling down over the edge of a cliff (or from a tight rope/balance beam).

D *From PD control law to control potential*

Reference [24] utilized a version of (2) that included only damping elements. Numerical instabilities and hip angle limitations were encountered when all six elements of the target linear and angular momentum vector were specified. This observation motivated the introduction of a selection matrix that limited the number of elements to be controlled to only some of those six. This problem was further addressed with leg "constraints," i.e. by tuning the predefined desired velocity for each foot [24,25]. We find this problem to be quite general —not every global forcing $(\vec{F},\vec{\tau})$ may be produced due to various physical limitations:

- ZMP is confined within the support base,
- limbs cannot penetrate other limbs or surrounding solid objects,
- joint angles and joint actuations are limited,
- ground friction coefficient is finite [26] etc.

If desired global forcing as suggested by (2) is outside the physically realizable region the simplest approach is to project the suggested solution to the physically realizable region. This approach was utilized in combination with prioritized control [8,19,9] for non-contact limb balancing [20]. A deficiency of the method is that with the ZMP at the boundary of the support base the smallest imprecision may destabilize the system. Yet another option is to use a method of control potentials.

The method of control potentials is particularly beneficial because a) physically meaningful solutions can be reinforced, as discussed below, and b) bias toward various target tasks can be easily introduced. The target task necessarily influences the choice of global forcing. For example, if the target task is to hold a glass of water, the whole body motion should not be very jerky.

We introduce the positive definite *Global state* control potential, V_{GS}, that may be either defined on $\left(\vec{F},\vec{\tau}\right)$ space with location of a minimum as suggested by the GMC PD law, (2),

$$V_{GS}(\vec{\tau},\vec{F}) = \left(\vec{\tau}-\vec{\tau}_{des.} \quad \vec{F}-\vec{F}_{des.}\right) \widetilde{V}_{GS} \begin{pmatrix} \vec{\tau}-\vec{\tau}_{des.} \\ \vec{F}-\vec{F}_{des.} \end{pmatrix}, \qquad \text{(5a)}$$

or it may be defined directly on $\vec{\theta}$, $\vec{L}(\vec{r}_{CM})$, $\vec{r}_{CM}$, $\vec{p}$ space as a sum of quadratic terms centered about the target values, i.e.

$$V_{GSX}(\vec{X}) = \left(\vec{X}-\vec{X}_{tar.}\right)\widetilde{V}_{GSX}\left(\vec{X}-\vec{X}_{tar.}\right), \quad V_{GS} = \sum_{X} V_{GSX} \,. \qquad \text{(5b)}$$

Although control potential (5a) and (5b) may be equally applicable we will assume (5a) in the rest of the manuscript. The formulation (5a) is also more compatible with time local control approach that we enforce, see III C. For practical purpose one may assume diagonal $\widetilde{V}_{GS}$ and introduce non-diagonal elements with two extra potentials described below.

E *The GMC on the global forcing space*

Here we outline the basic structure of the GMC framework. As illustrated in Fig. 2, the GMC potential defined on the global forcing $\left(\vec{F},\vec{\tau}\right)$ space may be represented as a sum of three control potentials:
- *Global State* convex potential (V_{GS}), (5a), minimized at the desired force and torque as suggested by (2);
- *Support base* potential (V_{SB}), enforcing ZMP within the support base and biased towards the innermost point, see Section III D;
- *Grand control* potential (V_G), enforcing other physical limitations due to the plant properties, particular plant state and environment as well as introducing bias toward the target tasks. The V_G represents a $\left(\vec{F},\vec{\tau}\right)$ projection of the *Joint control* potential (V_J) defined on joint space (see Section III E).

The final desired global forcing, $\left(\vec{F}^{final}_{des.},\vec{\tau}^{final}_{des.}\right)$, is then obtained by minimizing the unified GMC potential

$$V_{GMC} = V_{GS} + V_{SB} + V_G \qquad \text{(6)}$$

on $\left(\vec{F},\vec{\tau}\right)$ space. As we discuss in Section III C it may be beneficial to perform optimization directly on joint space instead of on smaller global forcing space.

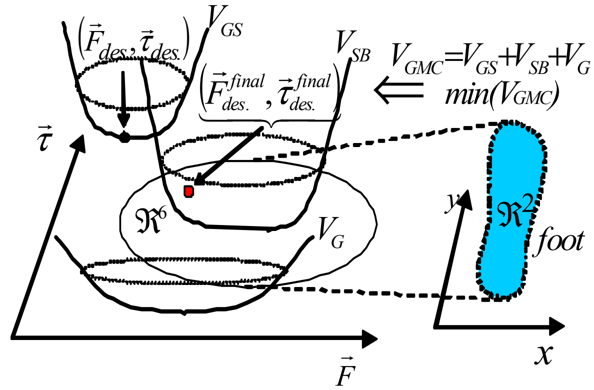

Fig. 2 The GMC potential defined on $\left(\vec{F},\vec{\tau}\right)$ space is a sum of *Global State* (V_{GS}), *Support Base* (V_{SB}), and *Grand* (V_G) potentials.

III. GMC IN JOINT SPACE

A *Physical Model of the world*

The performance of an on-line robotic controller depends on the physical model of the external world. Clearly, like a human, a robot has to tune the parameters of the physical model. However not all physical models are compatible with the tuning/adaptation schemes necessary for robotics applications. One example would be the model based on kinematical constraints. The kinematical constraints approach assumes an infinitely stiff ground and necessitates some of the GRF to be put by hand (only two contact points in 3-D may be resolved based on motion).

In contrast, the viscous-elastic approach is more natural as it assumes that interaction between the end-effector and external world, represented by Lagrange function, L, may be modeled as a function of the relative position and speed. Dynamics of the system are completely specified with 6 root Euler-Lagrange (E-L) equations and N_{joints} actuated joint angle E-L equations, or

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_{root}}\right)-\frac{\partial L}{\partial q_{root}}=\Gamma_{root}=0 \tag{7a}$$

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_{joints}}\right)-\frac{\partial L}{\partial q_{joints}}=\Gamma_{joints}\neq 0 \cdot \tag{7b}$$

This approach requires neither the position constraint equation nor the undetermined Lagrange multipliers required by the kinematical constraint approach. The interaction forces between body and external world are completely resolved by time local motion (i.e. state plus joint accelerations), i.e. they are not artificially assigned by the control designer. Finally, the characteristics of different surfaces can be modeled such that one can tell the difference between stepping on cement, deep snow, wet grass, or sandy beach.

B *Control flow*

As illustrated by Fig. 3 the knowledge of state (positions and velocities) and accelerations at time t_{n-1} defines the expectation of state at time t_n

$$\dot{\alpha}_{expected}(t_n)\approx\dot{\alpha}(t_{n-1})+\ddot{\alpha}(t_{n-1})(t_n-t_{n-1}) \tag{8}$$

$$\alpha_{expected}(t_n)\approx\alpha(t_{n-1})+\dot{\alpha}(t_{n-1})(t_n-t_{n-1})+\frac{1}{2}\ddot{\alpha}(t_{n-1})(t_n-t_{n-1})^2.$$

The state subsequently defines the expectation of forcing at time t_n. Given the net force and net torque the root segment (usually body trunk) accelerations may be expressed in terms of state and joint accelerations, see (7a), all defined at time t_n. Joint accelerations subsequently define unique joint torques at time t_n, see (7b), and state at time t_{n+1}, similar to (8).

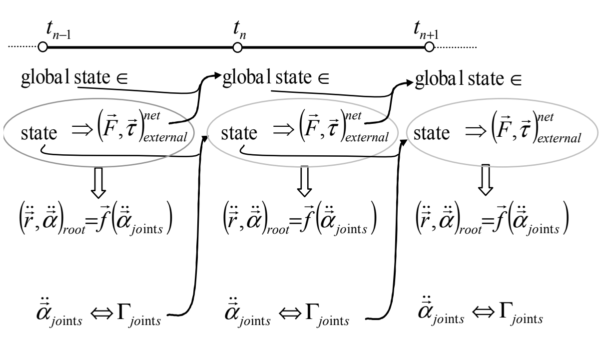

Fig. 3 Control flow.

The control task is to choose, based on complete information of state plus accelerations at time t_{n-1}, joint accelerations or equivalently joint torques at time t_n in order to define state and forcing at time t_{n+1}. The i[th] joint acceleration $\ddot{\alpha}_i(t_n)$ may be represented as the sum of joint acceleration $\ddot{\alpha}_i(t_{n-1})$ and joint jerk

$$\delta\ddot{\alpha}_i(t_n)=\ddot{\alpha}_i(t_n)-\ddot{\alpha}_i(t_{n-1}). \tag{9}$$

Hence, all quantities of interest at time t_{n+1} may be represented as a function of $\delta\ddot{\alpha}_i(t_n),\ i=1,\ldots,N_{joints}$.

C *Linear dynamics and simple cost function*

The expectation value of global kinematics quantities (CM position, CM momentum, whole body angular excursion and momentum about CM) at time t_{n+1} is independent of $\delta\ddot{\alpha}_i(t_n)$. The expectation of position, velocity, forcing of any end-effector and therefore net force is a linear in $\delta\ddot{\alpha}_i(t_n)$. Finally the expectation of moment about CM and principal angles (see Section II) is a quadratic function in $\delta\ddot{\alpha}_i(t_n)$.

The linear term, however, dominates for a small enough control time step (or small $\delta\ddot{\alpha}_i(t_n)$ as $\delta\ddot{\alpha}_i(t_n)_{\Delta t\to 0}\to 0,\forall i$). For small time step it makes sense to enforce linear dynamics for the optimizer by truncating higher order terms for all control variables.

The control problem may be even further simplified. Consider having only simple positive definite quadratic cost function terms

$$V=\sum_j\left(k_j-\sum_i l_{ji}\delta\ddot{\alpha}_i\right)^2 \tag{10}$$

where j counts different cost function terms and $i=1,\ldots,N_{joints}$. The minimum of V is expressed as a solution to a simple algebraic equation

$$\vec{K}=\vec{\vec{L}}\,\overrightarrow{\delta\ddot{\alpha}} \tag{11}$$

where $K_m=\sum_j k_j l_{jm}$ and $L_{mn}=\sum_j l_{jm}l_{jn}$.

The otherwise very complex and computationally demanding optimization problem on joint jerk space is now stated as a single algebraic equation. Because the solution can be obtained extremely quickly the control time step can be made very small to substantiate linear approximation. In contrast to hard-coded (priority list defined in advance by the control designer) prioritized control [8-9,19-20] this method is analytical and non-prioritized.

Another avenue would be to use general cost function terms, i.e. not truncated to linear terms in joint jerks, and apply a method that we call soft-coded prioritized control. The controller, local in time, may first satisfy the most sensitive task, defined by having largest overlap with gradient of control potential in $\delta\ddot{\alpha}_i(t_n)$ space, and then continue with next sensitive task in leftover space etc. If the cost function indeed properly communicates the nature of the problem then the priority list should reflect that in time-local fashion.

D *GMC Support Base control potential* V_{SB}

The ZMP as a function of the CM position, net force ($\vec{F} = M\vec{a}_{CM}$), and net moment about the CM can be expressed [15] as

$$x_{ZMP} = x_{CM} - \frac{F_x}{F_z + Mg} z_{CM} - \frac{\tau_y(\vec{r}_{CM})}{F_z + Mg} \qquad (12)$$
$$y_{ZMP} = y_{CM} - \frac{F_y}{F_z + Mg} z_{CM} + \frac{\tau_x(\vec{r}_{CM})}{F_z + Mg} .$$

We now introduce the positive definite *Support base* control potential, V_{SB}, defined on $(\vec{F}, \vec{\tau})$ space with the minimum corresponding to the innermost point of the support base (x^*_{zmp}, y^*_{zmp}), or

$$\begin{aligned} V_{SB}(\vec{\tau}, \vec{F}) &= V_{SB}\left(x_{zmp}(\vec{\tau}, \vec{F}), y_{zmp}(\vec{\tau}, \vec{F})\right) = \\ &V_{sb1}\left(\left(\vec{r}_{zmp} - \vec{r}^*_{zmp}\right) \cdot \vec{n}_1\right) \cdot V_{sb2}\left(\left(\vec{r}_{zmp} - \vec{r}^*_{zmp}\right) \cdot \vec{n}_2\right) \end{aligned}, \qquad (13)$$

where V_{sb1}, V_{sb2} are positive definite functions (reinforcing ZMP inside the support base) and $\vec{n}_1 \perp \vec{n}_2$ are unit eigenvectors of the *area matrix*

$$\vec{X}_{SB} = \int_{SB} \begin{bmatrix} (x - x^*_{zmp})^2 & (x - x^*_{zmp})(y - y^*_{zmp}) \\ (x - x^*_{zmp})(y - y^*_{zmp}) & (y - y^*_{zmp})^2 \end{bmatrix} dxdy \qquad (14)$$

and where integration is over the support base.

To summarize, the *Support Base* control potential, V_{SB}, penalizes net forcing when the ZMP is away from the innermost point. However, because this potential is only part of the GMC potential, the final choice of ZMP, while still physical, won't be at the innermost point of the support base.

E GMC *Joint control potential* V_J

The control designer may decide to include various terms in the *Joint potential*, V_J:

- End-effectors position/velocity/forcing
- Limits on joint angles
- Sum of joint torques squared (static energy criteria)
- Sum of joint powers (dynamic energy criteria) etc.

Although V_J is originally defined on $\delta\ddot{\alpha}$ space it may be useful to project it down to smaller $(\vec{F}, \vec{\tau})$ space. In this way one obtains *Grand potential*

$$V_G(\vec{F}, \vec{\tau}) = \min V_J(\delta\ddot{\alpha})|_{(\vec{F}, \vec{\tau})} \qquad (15)$$

as the minimum of V_J subject to $(\vec{F}, \vec{\tau})$ constraint. The motivation to study V_G clearly comes from the idea that GMC optimization may be performed on small $(\vec{F}, \vec{\tau})$ space alone. However if there is a large mismatch in dimensions between joint $\delta\ddot{\alpha}$ space and $(\vec{F}, \vec{\tau})$ space then (15) may represent a difficult optimization problem. One would need to introduce a control simplification like that described in Section III C. Then, however, the actual size of the space makes little difference and eventually all joint torques need be commanded.

To enforce no collisions between the end-effector and obstacle the control designer may use the attractive control potential term compatible with the analytical non-prioritized control approach, see Section III C. The attractive potential, however, should be such that the end-effector is attracted away from the obstacle. Still further, the magnitude of this term may be tuned to address the end-effector's speed.

IV GMC SUPPORT BASE PLANNING

A *Ground Phase Support Base Planning*

Here we propose a time-local control strategy for generation of the support base. This method is applicable when ground contact already exists and it is independent of other details (double vs. single support, left vs. right swing leg etc).

First we construct a *virtual GMC* potential,

$$V^{virtual}_{GMC} = V_{GS} + V_{G(J)} . \qquad (16)$$

Next we find a minimum of *virtual GMC* potential and obtain $\left(\vec{F}^{final}_{des.}, \vec{\tau}^{final}_{des.}\right)_{virtual}$. Using this virtual desired force and torque one may obtain a GMC-based virtual ZMP location on the ground. Now we propose that the virtual ZMP should be located at the innermost point of the virtual support base defined by the convex hull obtained by the feet projection onto ground [18], see Fig. 4a.. Therefore, indirectly, for the single support phase the virtual ZMP defines a target value for swing leg projection. For double support phase the virtual ZMP indirectly governs the feet rotation and toe-off.

Some simplification might be possible. For example, for some situations, one may neglect the *Joint* (or *Grand*) potential and use only *Global State* potential or the net forcing $\left(\vec{F}_{des.}, \vec{\tau}_{des.}\right)$ defined by the GMC PD law, (2), to obtain a simplified virtual ZMP. This approach is different from the one proposed in [14] where the zero moment condition is enforced.

The actual timing of ground contact of the swing leg may be forced to coincide with the expected time of the ZMP entering the "dangerous zone" near the edge of the stance footprint. This expected time may be obtained from the measured ZMP position and velocity.

Finally the swing leg velocity should be orthogonal to the surface at the moment of impact. The magnitude should be medium to avoid large stress.

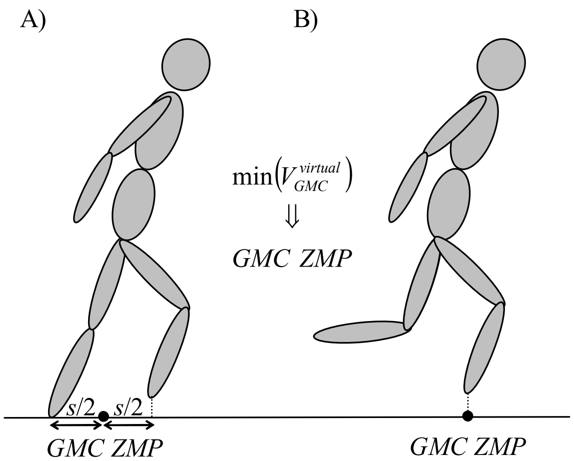

Fig. 4 GMC-based virtual ZMP during a) ground and b) flight phases.

B *Flight Phase Support Base Planning*

The GMC is only indirectly applicable for the aerial phase. The system's dynamics are then characterized by only the force of gravity and zero moment about the CM and are independent of suggested net forcing. Therefore the CM follows a parabolic trajectory with zero horizontal acceleration and the whole body's angular momentum is a conserved quantity. Furthermore, the control potential V_{SB} is undefined on $\left(\vec{F}, \vec{\tau}\right)$ space without ground contact. However the control potential V_{GS}, defined on $\left(\vec{F}, \vec{\tau}\right)$ space, and V_J, defined on $\delta\ddot{\alpha}$ space, still exist. As we argued, the GMC may be applied indirectly. The aerial phase optimizer may choose the landing time, placement of the foot and body posture/joint torque distribution that facilitate the subsequent GMC performance on the ground.

In the aerial phase, V_{GS} potential varies with time. Its minimum, defined by the GMC PD law, (2), may be used to define the desired position of the landing heel in the horizontal plane via (12), i.e $V_{GMC}^{virtual} \approx V_{GS}^{virtual}$. Alternatively, one may employ the minimum of $V_{GMC}^{virtual} = V_{GS} + V_{G(J)}$. The tendency to reposition the landing heel as a response to change in linear/angular position, vertical linear momentum and other joint cost functions may be represented by V_J potential with new cost function term. If landing time or position is given in advance, that should also be represented by appropriate weighting in V_J potential.

V SUMMARY AND FUTURE WORK

We propose a time-local optimization strategy where joint accelerations are varied to achieve whole-body dynamic postural balance. This strategy optimizes global body parameters such as spin angular momentum and body principal angles with respect to their equilibrium global state configuration as defined by the effective gravity vector. This optimization strategy is easily combined with other optimization objectives (e.g. maximal efficiency) subject to physical constraints such as requiring that the ZMP operates within the support base. We suggest, in parallel, two computational simplifications that may make the optimization problem in joint space tractable and easily implemented. We also propose a novel on-line strategy, founded on a GMC framework, addressing the problem of support base planning during ground and aerial locomotory phases.

In future investigations this theoretical model will be utilized and followed by several detailed studies on specific activities such as walking, running and jumping. These future investigations will simultaneously address human biomechanics and humanoid control. It is our hope that this work will lead to further investigation into online optimization techniques that address postural balance of legged systems, resulting in an even wider range of locomotory performance capabilities of legged robots and prostheses.

REFERENCES

[1] S. Schaal, "Is imitation learning the route to humanoid robots?" *Trends in Cognitive Sciences* 3:pp. 233-242, 1999.

[2] G. Pratt, "Legged Robots: What's New Since Raibert," *IEEE Robotics and Automation Magazine*. Research Perspectives. pp. 15-19, 2000.

[3] G. Pratt, "Low Impedance Walking Robots," *Integ. and Comp. Biol.* 42:pp. 174-181, 2002.

[4] A. Goswami, "Postural stability of biped robots and the foot-rotation indicator (FRI) point," *International Journal of Robotics Research* 18(6): pp. 523-533, 1999.

[5] B. Full and D. Koditschek, "Templates and Anchors: Neural Mechanical Hypotheses of Legged

Locomotion on Land," *J. Exp. Biol.* 202: pp. 3325-3332, 1999.
[6] D. Katic and M. Vukobratovic, "Survey of intelligent control techniques for humanoid robots," *Journal of Intelligent and Robotics Systems*, vol. 37, pp. 117-141, 2003.
[7] A. Goswami and V. Kallem, "Rate of change of angular momentum and balance maintenance of biped robots," *Proceedings of the IEEE International Conference on Robotics and Automation*, New Orleans, LA, U.S.A., pp. 3785-3790, 2004.
[8] O. Khatib, L. Sentis, J. H. Park and J. Warren, "Whole body dynamic behavior and control of human-like robots," *International Journal of Humanoid Robotics*, 1(1):pp. 29-43, March 2004.
[9] L. Sentis and O. Khatib, "Task-Oriented Control of Humanoid Robots Through Prioritization," *IEEE-RAS/RSJ International Conference on Humanoid Robots*, Santa Monica, USA, November 2004.
[10] G. A. Borelli. *De Motu Animalium* (English) Translated by P. Maquet, Springer-Verlag, Berlin, Heidelberg, 1989 [original version 1680].
[11] M. Vukobratovic and D. Juricic, "Contributions to the synthesis of biped gait," *IEEE Transactions on Biomedical Engineering*, BME-16, pp. 1-6, 1969.
[12] Q. Huang, K. Yokoi, S. Kajita, K. Kaneko, H. Aria, N. Koyachi and K. Tanie, "Planning walking patterns for a biped robot," *IEEE Transactions on Robotics and Automation*,17:(3) pp. 280-289, 2001.
[13] M. B. Popovic, W. Gu and H. Herr, "Conservation of Angular Momentum in Human Movement," *MIT AI Laboratory-Research Abstracts, September 2002*. pp. 231-232, 2002.
[14] M. B. Popovic, A. Hofmann and H. Herr, "Angular Momentum Regulation during human walking: Biomechanics and Control," *Proceedings of the IEEE International Conference on Robotics and Automation*, New Orleans, LA, U.S.A., pp. 2405-2411, April 2004.
[15] M. B. Popovic, A. Goswami and H. Herr, "Ground Reference Points in Legged Locomotion: Definitions, Biological Trajectories and Control Implications," *International Journal of Robotics Research* (Unpublished), 2005.
[16] M. B. Popovic and H. Herr, "Conservation of whole body angular momentum," Presented at the Neuro-Muscular Research Center Seminar Series. March 20, Boston University, Boston, Massachusetts, USA, March 2003.
[17] M. B. Popovic, A. Hofmann and H. Herr, "Zero spin angular momentum control: definition and applicability," *Proceedings of the IEEE-RAS/RSJ International Conference on Humanoid Robots*, Santa Monica, CA, November 2004.
[18] M. B. Popovic, A. Englehart and H. Herr, "Angular Momentum Primitives for Human Walking: Biomechanics and Control," *Proc. of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, Sendai, Japan., pp. 1685-1691, 2004.
[19] O. Khatib, K. Yokoi, K.-S. Chang, D. Ruspini, T. Holmberg and A. Casal, "Coordination and decentralized cooperation of multiple mobile manipulators," *Journal of Robotic Systems* 13(11):755-764, 1996.
[20] A. Hofmann, M. B. Popovic, S. Massaquoi and H. Herr, "A Sliding Controller for Bipedal Balancing Using Integrated Movement of Contact and Non-Contact Limbs," *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, Sendai, Japan, pp.1952-1959, 2004.
[21] H. Herr, G. P. Whiteley and D. Childress, "Cyborg Technology--Biomimetic Orthotic and Prosthetic Technology," *Biologically Inspired Intelligent Robots*, Bar-Cohen, Y. and C. Breazeal, Eds., SPIE Press, Bellingham, Washington, 2003.
[22] H. Herr and M. B. Popovic, "Angular momentum regulation in human walking,". *J. Exp. Biol.* (Unpiblished), 2005.
[23] T. Sugihara, Y. Nakamura and H. Inoue, "Realtime Humanoid Motion Generation through ZMP Manipulation based on Inverted Pendulum Control," *Proceedings of the IEEE International Conference on Robotics and Automation*, pp. 1404-1409, 2002.
[24] S. Kajita, F. Kanehiro, K. Kaneko, K. Fujiwara, K. Harada, K. Yokoi and H. Hirukawa, "Resolved Momentum Control: Humanoid Motion Planning based on the Linear and Angular Momentum," *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, Las Vegas, NV, pp. 1644-1650, 2003.
[25] S. Kajita, T. Nagasaki, K. Kaneko, K. Yokoi and K. Tanie "A Hop towards Running Humanoid Biped," *Proceedings of the IEEE International Conference on Robotics and Automation*, New Orleans, LA, pp. 629-635, 2004.
[26] S. Kajita, K. Kaneko, K. Harada, F. Kanehiro, K. Fujiwara and H. Hirukawa, "Biped walking on a low friction floor," *Proceedings of the IEEE/RSJ International Conference on Intelligent Robots and Systems*, Sendai, Japan., pp. 3546-3551, 2004.